Negative and fractional indices

Learning objective
To investigate the meaning of negative and fractional powers

Overview

This guided investigation establishes the rules for negative and fractional powers by looking at the tables of values for patterns.

Context

This activity can be used as the introduction to rational indices at GCSE, or as revision material at A level, leading to further algebraic work with rational indices.

Investigating resource

The investigation creates tables of values and then uses number patterns to establish the definitions of negative powers. It uses tables of values for fractional powers to understand what they do. It leads to an algebraic definition for rational indices.

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From [MENU] choose **Table**. Choose **SET** [F5] and input Start 1, End 4 and Step 1, using [EXE] to save your settings.

Task

1. Type $Y1 = 2^x$ to create a table of values for powers of 2 from 1 to 4.
   a. Write the values on the shaded part of the table below.
   b. Write down the pattern in the answers as you go up the table (x getting smaller).
   c. Use the pattern to predict the values of 2^x when $x = 0$ and when x is negative.
   d. Change the settings so the table of values shows negative values of x and check your answers. (Highlight each decimal answer and press [S⇔D] to show fractional values.)
2. Repeat for the other numbers – is there a similar pattern?
3. How could you express this algebraically?

x	$Y1 = 2^x$	$Y2 = 3^x$	$Y3 = \left(\frac{1}{2}\right)^x$	$Y4 = \left(\frac{3}{2}\right)^x$
-3	$2^{-3} =$	$3^{-3} =$		
-2	$2^{-2} =$	$3^{-2} =$		
-1	$2^{-1} =$	$3^{-1} =$		
0	$2^0 =$	$3^0 =$		
1	$2^1 =$	$3^1 =$		
2	$2^2 =$	$3^2 =$		
3	$2^3 =$	$3^3 =$		
4	$2^4 =$	$3^4 =$		

b. ....

....

....

Expressing the pattern algebraically $a^{-n} =$ ....

....

Follow-up

Write down fractional answers to the following. Choose **Run-Matrix** mode to check the answers.

a. $2^{-4} =$ b. $5^{-1} =$ c. $\left(\frac{2}{7}\right)^0 =$ d. $\left(\frac{3}{5}\right)^{-2} =$ e. $\left(\frac{1}{4}\right)^{-3} =$ f. $\left(\frac{2}{7}\right)^{-2} =$

CASIO

Introduction to fractional powers

Investigate 1

Create a table of values for $Y1 = x^{\frac{1}{2}}$ for values of x from 0 to 50.

a. Write down the rows of the table which give whole number values.

b. Write down what you notice.

c. What does the power $\frac{1}{2}$ achieve? Write your answer algebraically.

Repeat for $Y2 = x^{\frac{1}{3}}$. Summarise your findings.

x	$Y1 = x^{\frac{1}{2}}$

x	$Y2 = x^{\frac{1}{3}}$

x	$Y3 = x^{\frac{3}{2}}$

x	$Y4 = x^{\frac{2}{3}}$

b. ....

....

c. ....

Summary: ....

Investigate 2

1. Repeat the investigation above for $Y3 = x^{\frac{3}{2}}$ and $Y4 = x^{\frac{2}{3}}$
2. Explain how the two parts of the fraction work together to give your results.

Explanation: ....

....

Follow-up 2

Use your rules to find answers to the following. Use **Run-Matrix** mode if you need to, but think how you could have worked it out without a calculator.

a. $81^{\frac{1}{2}} =$ b. $\left(\frac{1}{25}\right)^{-\frac{1}{2}} =$ c. $\left(\frac{25}{64}\right)^{\frac{1}{2}} =$ d. $\left(\frac{36}{25}\right)^{-\frac{1}{2}} =$ e. $125^{\frac{2}{3}} =$ f. $\left(\frac{125}{216}\right)^{-\frac{2}{3}} =$

Aims

The investigation is designed to work from calculated values of powers to a general understanding written algebraically.

The task also reinforces the need for accuracy when entering calculations.

Resources

Supporting resources are available in our Resource Centre.

Before the lesson

How-To Videos

- Calculation and Reset – Getting Started
- Table – Calculating Values

Learner support material

- Indices and surds

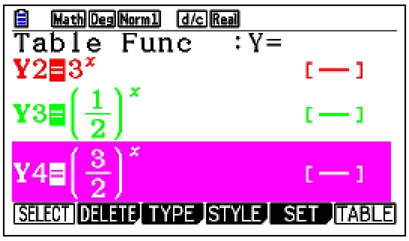
During the lesson

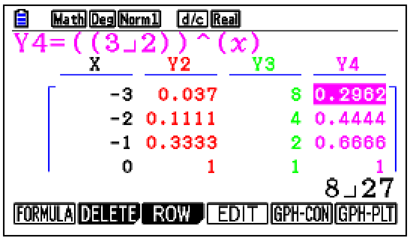
- Learner worksheet

Using the fx-CG50

Most of the skills required for this activity are covered in the skills videos listed above.

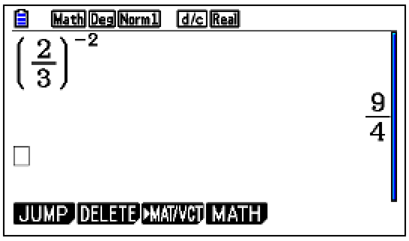
Additional functions can be added and displayed as new columns on the table.

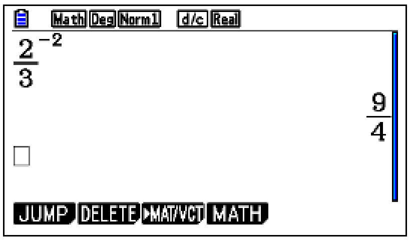




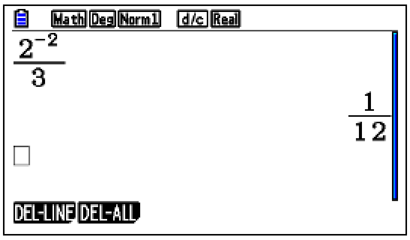
Dealing with the unexpected

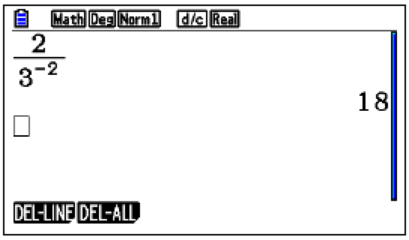
Learners may not realise that they have typed their expressions incorrectly when fractions are involved. To find the fraction $\frac{2}{3}$ all to the power -2, you need $\left(\frac{2}{3}\right)^{-2}$ The calculator will give the correct answer without brackets, providing you scroll to the right after entering the fraction before entering the power.

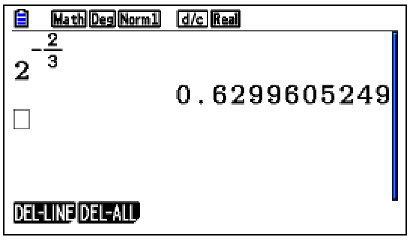




The expressions above are both correct, but the ones below are different calculations that learners may use in error.







Prompts

Ask learners to think about whether brackets are needed as they input their calculations.

Ask learners to think how the columns for 2^x and $\left(\frac{1}{2}\right)^x$ are related.

Ask learners to think how the columns for 3^x and $\left(\frac{1}{2}\right)^x$ can be combined to give the column for $\left(\frac{3}{2}\right)^x$.

Ask whether negative powers ever give negative numbers.

Extension questions

Ask learners to think about the laws of indices – they can deduce the definitions from the laws.

- Multiplying a^n by 1 does not change the number. So, writing 1 as a power of a must give the power of a to be zero as it does not change the power when added.
- Multiply a^n by a^{-n}. What happens to the answer? What happens to the powers?
- Raising a^n to the power $\frac{1}{n}$ gives a^1. How does that imply that raising a number to the power $\frac{1}{n}$ is the same as finding the n^{th} root?

Ask learners to investigate whether you get the same answer if you square first and then cube root, and when you do the same two operations in the opposite order. Which is easier?

Task

x	$Y1 = 2^x$		$Y2 = 3^x$		$Y3 = \left(\frac{1}{2}\right)^x$	$Y4 = \left(\frac{3}{2}\right)^x$
-3	$2^{-3} =$	$\frac{1}{8} = \frac{1}{2^3}$	$3^{-3} =$	$\frac{1}{27} = \frac{1}{3^3}$	8	$\frac{8}{27}$
-2	$2^{-2} =$	$\frac{1}{4} = \frac{1}{2^2}$	$3^{-2} =$	$\frac{1}{9} = \frac{1}{3^2}$	4	$\frac{4}{9}$
-1	$2^{-1} =$	$\frac{1}{2}$	$3^{-1} =$	$\frac{1}{3}$	2	$\frac{2}{3}$
0	$2^0 =$	1	$3^0 =$	1	1	1
1	$2^1 =$	2	$3^1 =$	3	$\frac{1}{2}$	$\frac{3}{2}$
2	$2^2 =$	4	$3^2 =$	9	$\frac{1}{4}$	$\frac{9}{4}$
3	$2^3 =$	8	$3^3 =$	27	$\frac{1}{8}$	$\frac{27}{8}$
4	$2^4 =$	16	$3^4 =$	81	$\frac{1}{16}$	$\frac{81}{16}$

1b. Divide by 2 as you go up the column for 2^x.

3. $a^{-n} = \dfrac{1}{a^n}$

Follow-up

a. $2^{-4} = \frac{1}{16}$ b. $5^{-1} = \frac{1}{5}$ c. $\left(\dfrac{2}{7}\right)^0 = 1$ d. $\left(\dfrac{3}{5}\right)^{-2} = \dfrac{25}{9}$ e. $\left(\dfrac{1}{4}\right)^{-3} = 64$ f. $\left(\dfrac{2}{7}\right)^{-2} = \dfrac{49}{4}$

Investigate 1

x	$Y1 = x^{\frac{1}{2}}$
0	0
1	1
4	2
9	3
16	4
25	5
36	6
49	7

x	$Y2 = x^{\frac{1}{3}}$
0	0
1	1
8	2
27	3

x	$Y3 = x^{\frac{3}{2}}$
0	0
1	1
4	8
9	27
16	64
25	125
36	216
49	343

x	$Y4 = x^{\frac{2}{3}}$
0	0
1	1
8	4
27	9

b. The square numbers give their square root when raised to the power a half.

c. Power a half is the same as square root

Summary: Power one third is cube root, so it makes sense that power $\frac{1}{n}$ is the same as nth root.

Investigate 2

Explanation: The denominator of the power gives the root and the numerator raises the answer to that power – these things can be done in either order.

Follow-up 2

a. $81^{\frac{1}{2}} = 9$ b. $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = 5$ c. $\left(\frac{25}{64}\right)^{\frac{1}{2}} = \frac{5}{8}$ d. $\left(\frac{36}{25}\right)^{-\frac{1}{2}} = \frac{5}{6}$ e. $125^{\frac{2}{3}} = 25$ f. $\left(\frac{125}{216}\right)^{-\frac{2}{3}} = \frac{36}{25}$