Calculating probabilities using the binomial distribution

| Learning objectives |
|---|
| How to calculate probabilities and cumulative probabilities using the binomial distribution; |
| Using the inverse binomial distribution and Table mode to find critical regions for a hypothesis test. |

This worksheet accompanies the video *Binomial Distribution Probability and Hypothesis Testing*. We suggest you watch it first.

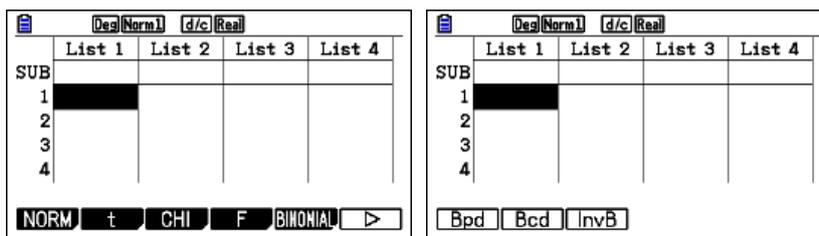## How to use the distribution menu in Statistics mode for the binomial distribution

Press MENU and choose **Statistics**.

Choose **DIST** (F5).
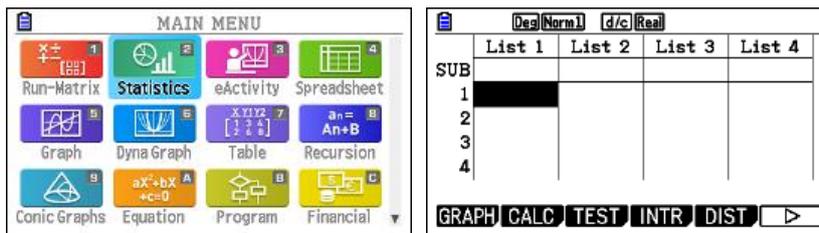
In this example, the random variable X is taken from a binomial distribution with 10 trials and probability 0.6 of success. Find $P(X = 7)$ when $X \sim B(10, 0.6)$.
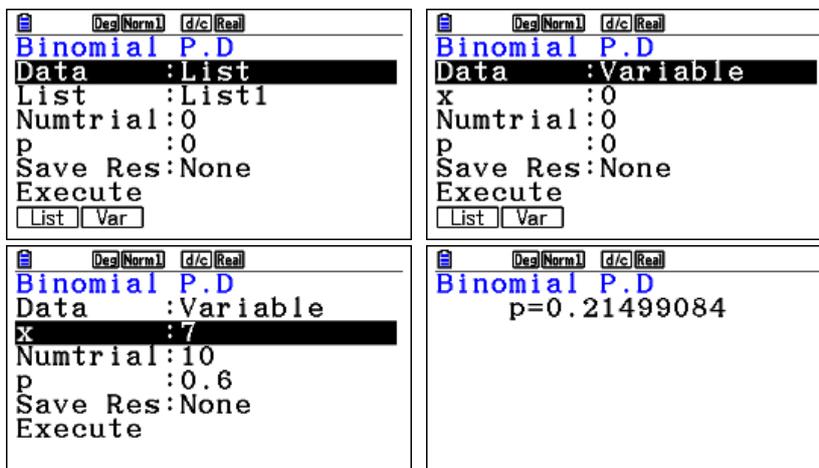
Choose **BINOMIAL** (F5) followed by **Bpd** (F1).

For Data, choose **Var** (F2).

Enter the value of x and the parameters Numtrial: (*n* = 10) and p: 0.6.

Press EXE .

Press EXE again to return to the previous screen and change the value of x as required.

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Finding cumulative probabilities using the distribution menu in Statistics mode

In this example, the random variable X is taken from a binomial distribution with 10 trials and probability 0.6 of success. Find $P(X \leq 7)$ when $X \sim B(10, 0.6)$.

Return to the Statistics main screen by pressing [EXIT] twice.

Choose **DIST**, **BINOMIAL** and **Bcd.**

Choose **Var**, the range of values of x, and the parameters Numtrial: (*n* = 10) and p: 0.6. Press [EXE] .

Press [EXE] again to return to the previous screen and change the value of x as required.

How to use trial and error for cumulative probabilities for a two-tail hypothesis test

In this example, the null hypothesis states that the probability of success is 0.5 and there are 50 trials. We wish to find the critical region for a two tailed test at the 10% significance level.

$$H_0 : p = 0.5 \quad H_1 : p \neq 0.5$$

Press [EXIT] twice again to return to the main Statistics screen.

Choose **DIST** ([F5]), followed by **BINOMIAL** and **Bcd**.

We need a cumulative probability at each end of 5%.

Check **Var** is chosen and set the parameters Numtrial: (n = 50) and p: 0.5.

For the left tail, keep the Lower value of X at 0 and change the Upper value until you have seen values either side of 5% (0.05).

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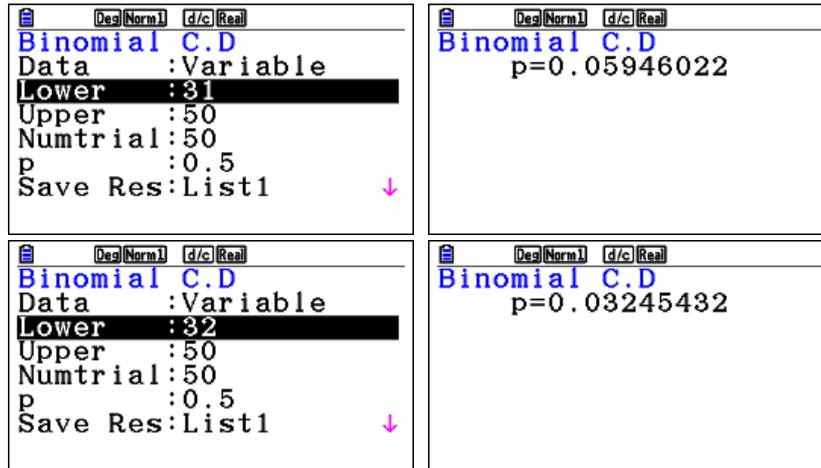
At the right tail, input the Upper value at 50 and change the Lower value until you have seen values either side of 5% (0.05)


The values of X which form the critical regions are 0 to 18 and 32 to 50 inclusive.

Notice that these are symmetrically placed either side of 25, the expected number of heads. In cases where the probability is not 0.5, the tails can contain very different number of values for equal probabilities.

### Taking it further – showing a list of cumulative probabilities in Table mode

Critical regions are easier it you have a list of cumulative binomial probabilities similar to those published in statistical tables, $P(X \le x)$. You can also create a list of probabilities for regions at the top end $P(X \ge x)$ from which you can easily see the critical region at the top end.
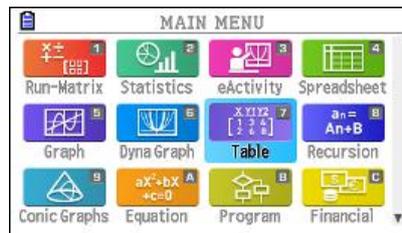
Press MENU and choose **Table**.

To find the binomial cumulative probabilities, press OPTN and choose the further menu ▷ (F6) for the second set of menu choices.

From here, choose **STAT** (F3) followed by **DIST** (F1), **BINOMIAL** (F5) and then **Bcd** (F2).
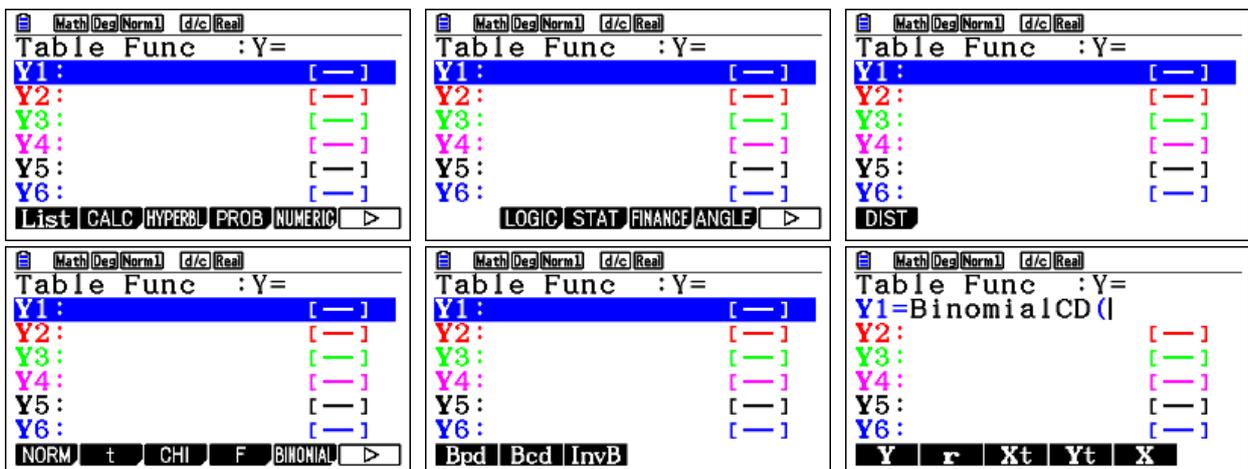
For Y1, enter the information in the following order – lower value 0, upper value x (using the [X,θ,T] key for x), the number of trials 50 and the probability of success 0.5, each separated by a comma. (If you forget the order, check how Statistics mode asks for the values – it the same order here.)

Close the bracket and press [EXE] .

For Y2, repeat the same steps as above, but setting lower value x , upper value 50, the number of trials 50 and the probability of success 0.5, each separated by a comma.
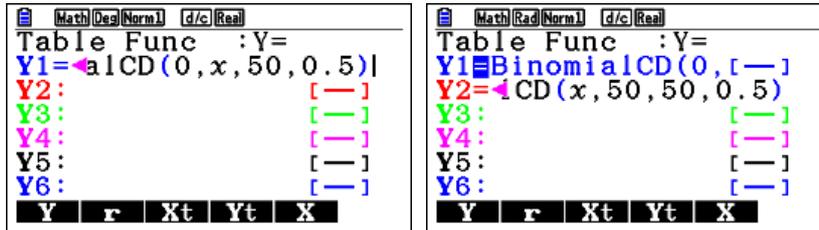
To create a table for all the values, choose **SET** and input Start: 0, End: 50 and Step: 1.

Press [EXE] to enter the values and choose **TABLE** ([F6]) to see the table of values.

Scroll across to see each value to more decimal places and down to see other rows on the table.
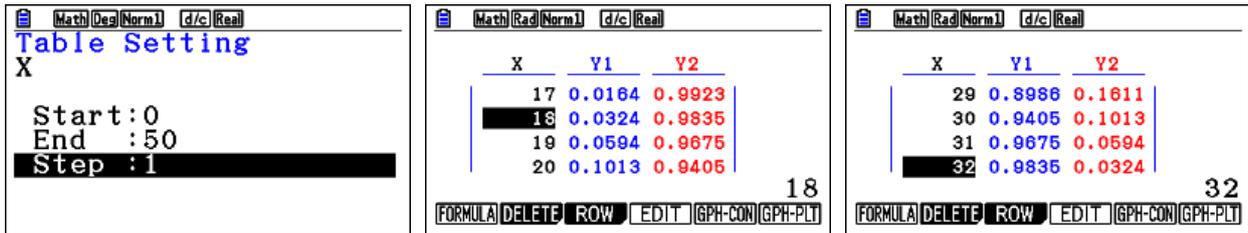
Notice that the values on each row add up to more than 1, as the probability of taking the X value is included in both sets. The value in Y1 and the Y2 from the row below add up to 1.

Scroll down until you see where the probabilities Y1 change from below 5% to above 5% and at the top from above 5% to below 5%. So, the lower tail of the critical region is {0, 1, 2, … 18} and the upper tail consists of values above 31 so {32, 33, … 50}.
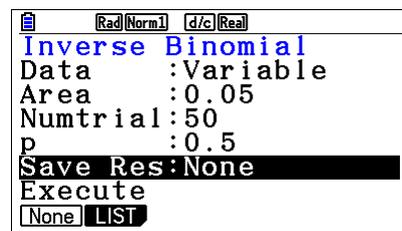
Taking it further – using inverse Binomial probabilities
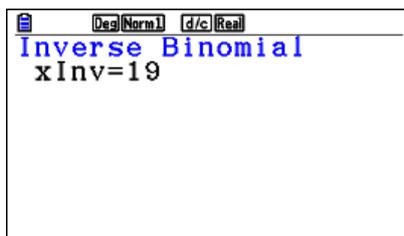
Press [MENU] and choose **Statistics**.

Choose **DIST** followed by **BINOMIAL** and **InvB** ([F3]).
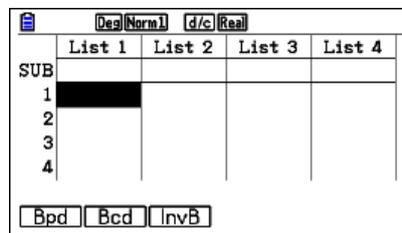
To find the critical values at the lower end, set the value of Area to 0.05, and set the parameters Numtrial: (n = 50) and p: 0.5.

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Press EXE .

This is the value of X for which the probability is closest to 0.05 – in this case it is the one for which the probability is just over 0.05 – so it is important to find the actual probability $P(X \leq 19)$ before deciding which values lie in the critical region. It is important to record the probabilities either side of 0.05 in your written answer as evidence that your answer is correct.

For the other tail, select an area of 0.95 and press EXE .

A warning will appear as the probability $P(X \leq 30)$ is more than 0.01 away from the 0.95 that we are looking for.
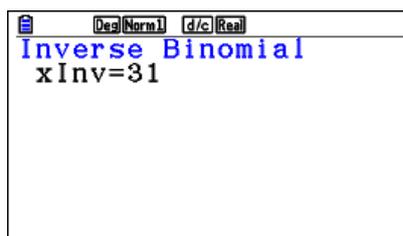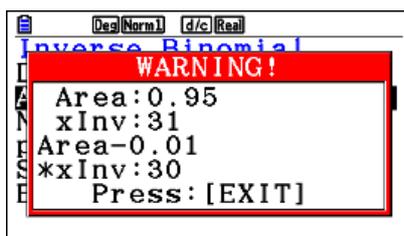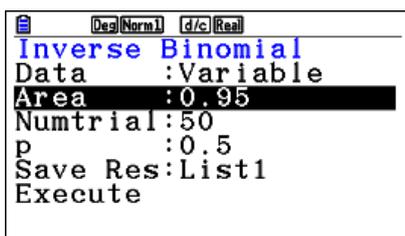
Again, it is necessary to check the exact values of probability before deciding which values lie in the critical region. It is important to record the probabilities either side of 0.05 in your written answer as evidence that your answer is correct for both tails.