

# The constant acceleration equations

## Learning objective

To practise solving the linear, quadratic and simultaneous equations arising from the SUVAT equations

To understand how to translate wordy questions into routine algebra

## Overview

The resource begins with algebra practice using the constant acceleration equations allowing learners to build their skills to solve linear, quadratic and simultaneous equations that arise. It can be used as an introduction to the calculator functionality in **Equation** including substitution into formulae using **SOLVER**. The resource includes problem solving using the constant acceleration equations.

## Context

This can be used as an introduction to the use of the constant acceleration equations once the learners have seen their derivation. The worksheet can be completed using traditional algebraic methods or the calculator functionality.

## Teaching resource

The worksheet consists of two exercises – one where the values of some of the variables are given and others are to be found. The second exercise has wordier problems to be translated into “when these variables have these values, find this”. Guidance is given on how to select the correct equation for an efficient method by looking for the “missing” variable.

## Contents

Learner worksheet – pages 2-3

Learner instructions for using SOLVER – pages 4-5

Teaching notes – page 6

Aims

Resources

Using the fx-CG50

Dealing with the unexpected

Prompts

Extension questions

Solutions – page 7

Learning resource objective

Be able to solve equations arising from the SUVAT equations

To translate wordy problems into routine algebra

The equations of motion for a particle moving with constant acceleration are

$$s = \frac{1}{2}(u + v)t$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

where  $s$  is the displacement from the original position,  $u$  is the initial velocity,  $v$  is the final velocity,  $a$  is the acceleration (assumed constant) and  $t$  is the time taken.

For each equation write which variable is “missing” from the equation. Use this to select the equation to use for each problem.

Complete the table below. All quantities are in standard SI units.

	Given that ...	Calculate	Missing ...	Use the equation ...	Answer
1	$u = 0, a = 1.7, t = 10$	$v$			
2	$s = 65, a = 0.2, t = 10$	$u$			
3	$s = 30, u = 8, v = -4$	$t$			
4	$u = -5, v = 7, a = 0.8$	$s$			
5	$u = -4, v = 5, a = 1.5$	$t$			
6	$s = 60, v = 6, t = 8$	$a$			
7	$s = 20, u = 10, t = 5$	$v$			
8	$s = 5, v = 0, a = -0.4$	$u$			
9	$s = 2, u = 17, a = -1.9$	$t$			
10	$s = 5.5, t = 1$ and $s = 52, t = 4$	$u$ $a$			

## Problem solving

List the variables you know, what you need to find and the missing variable. Choose your equation, substitute the given values and solve.

1. A cyclist travels 50m. She accelerates uniformly from  $3 \text{ ms}^{-1}$  to  $8 \text{ ms}^{-1}$ . Calculate how long she is travelling.

.....  
.....  
.....  
.....

2. A car accelerates from rest to  $7 \text{ ms}^{-1}$ . It travels 10 m. Calculate the acceleration of the car assumed constant.

.....  
.....  
.....  
.....

3. A ball is thrown into the air and reaches its highest point after 3s. How high is this above the point of projection? (Hint: At the highest point, the velocity of the ball is zero.)

.....  
.....  
.....  
.....

4. A train slows from  $45 \text{ ms}^{-1}$  to  $5 \text{ ms}^{-1}$  uniformly in 1 minute 20 seconds. Calculate the acceleration of the train.

.....  
.....  
.....  
.....

5. A pebble is thrown vertically upwards at  $18 \text{ ms}^{-1}$ . At what times is it 10m above the ground?

.....  
.....  
.....  
.....

6. A lorry passes a road-sign. Two seconds later the lorry is at a point A 9.2 m from the sign. Four seconds after that the lorry is at a point B which is 66 m from the road-sign. Calculate the velocity of the lorry as it passes the road-sign and the acceleration of the lorry assumed constant. (Hint: consider the overlapping journeys from the road-sign to A and from the road-sign to B.)

.....  
.....  
.....  
.....

## Learning resource objective

Be able to use **SOLVER** to substitute values into an equation to solve it

To use **POLY** to solve a quadratic equation

To use **SIMUL** to solve simultaneous equations

From **MENU** choose **Equation** and **SOLVER** (**F3**). You can use this in two ways. Firstly, type any equation using the variable key **X,θ,T** for an unknown in any type of equation. Secondly, type a formula and specify the values of all except one of the letters in the formula, and solve for the remaining variable.

### Example 1

To find the value of **V** when **U = 0**, **A = 1.7** and **T = 10**.

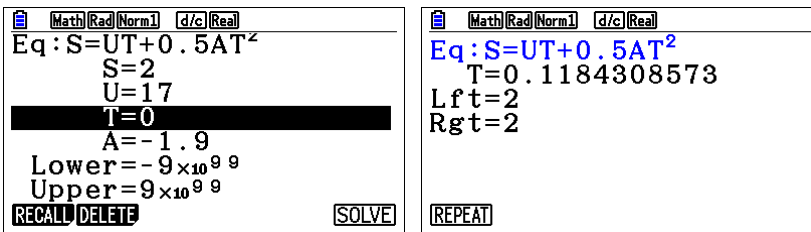
Type the formula using **ALPHA** - **2** for **V** and **SHIFT** - **=** for the equals sign. Pressing **EXE** will bring up a list of the variables, each of which will have the value of that variable the previous time it was used. The default settings are zero for each. Scroll to each of the known variables, type its value and press **EXE**. Scroll back up to highlight the variable to be found and choose **SOLVE** (**F6**).

### Example 2

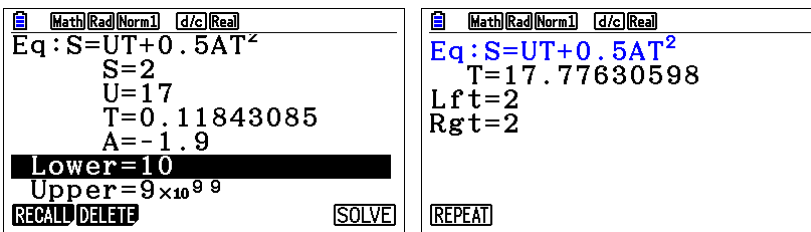
The variable does not have to be the subject of the formula to be found. To find the value of **T** when **V = 5**, **U = -3** and **A = 1.5** in the same equation, choose **REPEAT** (**F1**) and change the values of the variables in the list. To find **T**, scroll to **T** (and ignore the value that is still showing from the previous calculation) before choosing **SOLVE** (**F6**).

Example 3

**SOLVER** can be used to find  $t$  when  $s=2$ ,  $u=17$ ,  $a=-1.9$  without rearranging the equation  $s = ut + \frac{1}{2}at^2$ , but it will only find one root and surd answers are not given. Where there are two roots, it is not clear in advance which one the method will find.

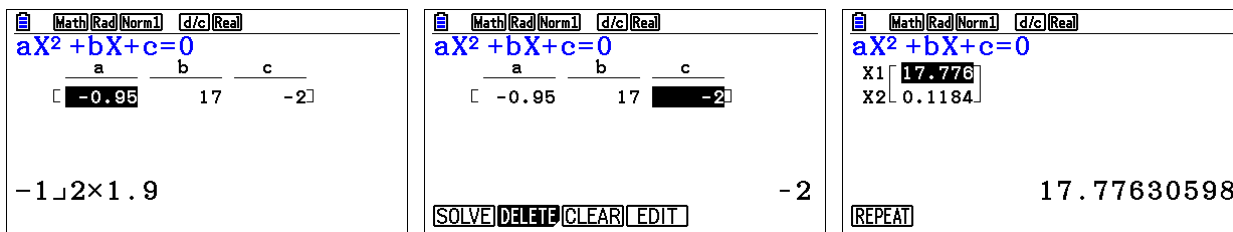


To find the second one, change the range in which the root is to be found, and the other may be given.



When the equation is quadratic in  $t$ , it is better to press **EXIT** to leave **SOLVER** and choose **POLY** (**F1**) followed by **Degree: 2** (**F1**). Rearrange into the standard arrangement  $ax^2 + bx + c = 0$  and enter the coefficients – you do not have to work them out before entering.

$2 = 17t + \frac{1}{2}(-1.9)t^2$  rearranges to  $-\frac{1}{2} \times 1.9t^2 + 17t - 2 = 0$



Example 4

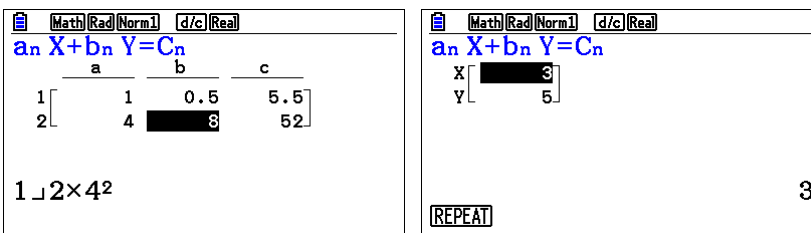
To solve the simultaneous equations

$5.5 = u + \frac{1}{2}a$

$52 = 4u + 8a$

Make sure that the equations are arranged in the standard arrangement.

Use **EXIT** to go back and choose **SIMUL** (**F1**) with **Number of Unknowns: 2** (**F1**). Enter the coefficients, which can be entered as calculations rather than values, and choose **SOLVE** (**F1**). Take care with which of the values is  $u$  and which is  $a$  as the calculator will call them  $X$  and  $Y$ .



## Aims

To be proficient in using the calculator functionality to solve equations including quadratic and simultaneous equations

## Resources

Supporting resources are available in our Resource Centre.

*Before the lesson*

How To Videos

- Quadratic equations – Solving Numerically
- Simultaneous equations – Solving Numerically

Learner support material

- SUVAT Equations Confidence Builder

*During the lesson*

- Learner worksheet
- Learner sheet for using the fx-CG50

*After the lesson*

- Variable acceleration classroom resource

## Using the fx-CG50

Full instructions are given in the learner resource sheet on pages 4-5.

## Dealing with the unexpected

Whenever an equation is entered into **SOLVER**, the list of variables is already populated with previously used values for **A**, **T**, etc. If you do not scroll to the value to be found before pressing **SOLVE**, then the calculator will use the value of the variable that is currently stored, which might be zero, and solve for that. Learners may then complain that the calculator has changed the values they have typed in!

When using **SOLVER** for quadratic equations, changing the **Lower** or **Upper** limit of the interval to force the calculator to find a second root does not always produce the intended value. You might need several guesses to find a value that works.

## Prompts

To remember the equations, remember that they are all + except for  $s = vt - \frac{1}{2}at^2$  which has a “working backwards” feel about it. (You may find it better only to teach 4 equations, and omit this one altogether.)

To select the efficient method, prompt learners to look at which variables they know, which they need to find and which is left out – choose the equation with the correct “missing” variable.

Prompt learners to use the calculator functionality at least as a checking strategy.

When solving quadratic equations, errors are sometimes made at the rearranging stage. A good checking strategy is to write down the rearranged version and the answers from **POLY**, and also to use **SOLVER** for the original version of the equation. If the root found in **SOLVER** is the same as one of the roots from the rearranged equation, it is very likely to be a correct rearrangement.

## Extension questions

- 1) When might the SUVAT equations model a real-life situation well?
- 2) Why might they only be valid for relatively small values of  $t$ ?

Solutions

	Given that ...	Use the equation ...	Calculate ...
1	$u = 0, a = 1.7, t = 10$	$v = u + at$	$v = 17$
2	$s = 65, a = 0.2, t = 10$	$s = ut + \frac{1}{2}at^2$	$u = 5.5$
3	$s = 30, u = 8, v = -4$	$s = \frac{1}{2}(u + v)t$	$t = 15$
4	$u = -5, v = 7, a = 0.8$	$v^2 = u^2 + 2as$	$s = 15$
5	$u = -4, v = 5, a = 1.5$	$v = u + at$	$t = 6$
6	$s = 60, v = 6, t = 8$	$s = vt - \frac{1}{2}at^2$	$a = -0.375$
7	$s = 20, u = 10, t = 5$	$s = \frac{1}{2}(u + v)t$	$v = -2$
8	$s = 5, v = 0, a = -0.4$	$v^2 = u^2 + 2as$	$u = \pm 2$
9	$s = 2, u = 17, a = -1.9$	$s = ut + \frac{1}{2}at^2$	$t = 0.117$ or $17.8$
10	$s = 5.5, t = 1$ and $s = 52, t = 4$	$s = ut + \frac{1}{2}at^2$	$u = 3$ $a = 5$

Problem solving

	Given that ...	Use the equation ...	Calculate ...
1	$s = 50, u = 3, v = 8$	$s = \frac{1}{2}(u + v)t$	$t = 22.7$ s
2	$s = 10, u = 0, v = 7$	$v^2 = u^2 + 2as$	$a = 2.45$ ms <sup>-2</sup>
3	$v = 0, a = -9.8, t = 3$	$s = vt - \frac{1}{2}at^2$	$s = 44.1$ m
4	$u = 45, v = 5, t = 80$	$v = u + at$	$a = 0.5$ ms <sup>-2</sup>
5	$s = 10, u = 18, a = -9.8$	$s = ut + \frac{1}{2}at^2$	$t = 0.682$ and $2.99$ s
6	$s = 9.2, t = 2$ and $s = 66, t = 6$	$s = ut + \frac{1}{2}at^2$	$u = 1.4$ ms <sup>-1</sup> $a = 3.2$ ms <sup>-2</sup>

Extension questions

- 1) The SUVAT equations model a real-life situation well when the forces acting on a body are constant. For acceleration produced by an engine with a constant power output, the force will decrease as the velocity increases.
- 2) For very large values of  $t$  the velocities will exceed sensible values for the situation.